3.2 Use Parallel Lines and Transversals

Before You identified angle pairs formed by a transversal.

Now You will use angles formed by parallel lines and transversals.

Why? So you can understand angles formed by light, as in Example 4.

Key Vocabulary

- **corresponding angles,** *p. 149*
- **alternate interior angles,** *p. 149*
- **alternate exterior angles,** *p. 149*
- **consecutive interior angles,** *p. 149*

ACTIVITY Explore Parallel Lines

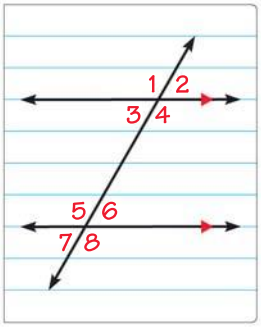
Materials: lined paper, tracing paper, straightedge

STEP 1 **Draw** a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.

STEP 2 **Trace** your drawing onto tracing paper.

STEP 3 **Move** the tracing paper to position $\angle 1$ of the traced figure over $\angle 5$ of the original figure. Compare the angles. Are they congruent?

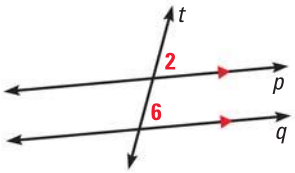
STEP 4 **Compare** the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal?



POSTULATE For Your Notebook

POSTULATE 15 Corresponding Angles Postulate

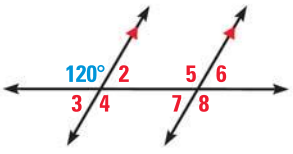
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



$\angle 2 \cong \angle 6$

EXAMPLE 1 Identify congruent angles

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.



Solution

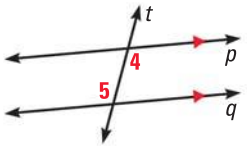
By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because $\angle 4$ and $\angle 8$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.

THEOREMS *For Your Notebook*

THEOREM 3.1 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Proof: Example 3, p. 156

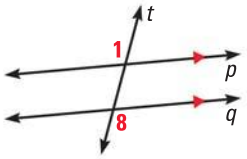


$\angle 4 \cong \angle 5$

THEOREM 3.2 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Proof: Ex. 37, p. 159

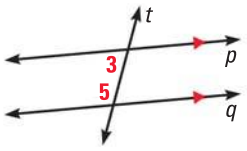


$\angle 1 \cong \angle 8$

THEOREM 3.3 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

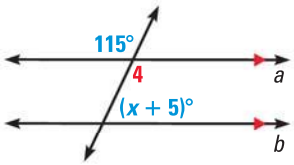
Proof: Ex. 41, p. 159



$\angle 3$ and $\angle 5$ are supplementary.

EXAMPLE 2 Use properties of parallel lines

ALGEBRA Find the value of x.



Solution

By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

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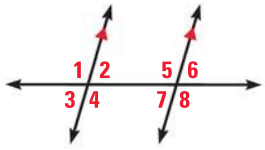
GUIDED PRACTICE for Examples 1 and 2

Use the diagram at the right.



1. If $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which postulate or theorem you use in each case.
2. If $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.

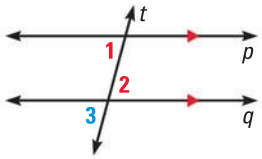
EXAMPLE 3 Prove the Alternate Interior Angles Theorem

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

WRITE PROOFS
You can use the information from the diagram in your proof. Find any special angle pairs. Then decide what you know about those pairs.

Solution

Draw a diagram. Label a pair of alternate interior angles as $\angle 1$ and $\angle 2$. You are looking for an angle that is related to both $\angle 1$ and $\angle 2$. Notice that one angle is a vertical angle with $\angle 2$ and a corresponding angle with $\angle 1$. Label it $\angle 3$.



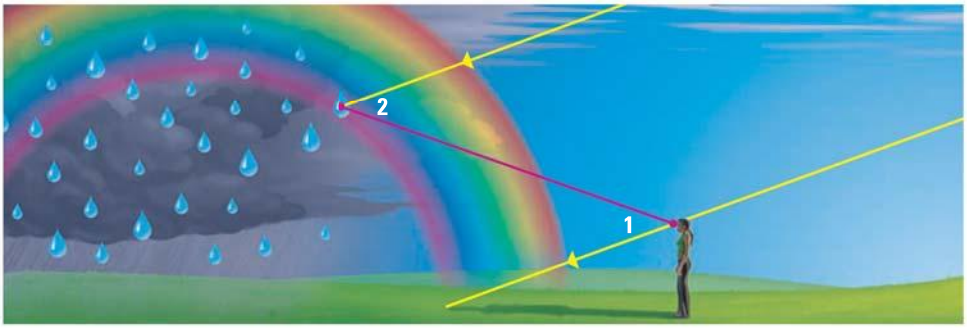
GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence

EXAMPLE 4 Solve a real-world problem

SCIENCE When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?



Solution

Because the sun's rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

GUIDED PRACTICE for Examples 3 and 4

3. In the proof in Example 3, if you use the third statement before the second statement, could you still prove the theorem? *Explain*.

4. **WHAT IF?** Suppose the diagram in Example 4 shows yellow light leaving a drop of rain. Yellow light leaves the drop at an angle of 41°. What is $m\angle 1$ in this case? How do you know?

3.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 21, 33, 39, and 40

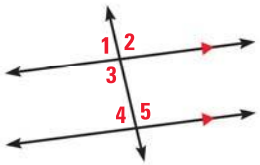
SKILL PRACTICE

EXAMPLES 1 and 2 on pp. 154–155 for Exs. 3–16

1. **VOCABULARY** Draw a pair of parallel lines and a transversal. Label a pair of *corresponding angles*.

2. ★ **WRITING** Two parallel lines are cut by a transversal. Which pairs of angles are congruent? Which pairs of angles are supplementary?

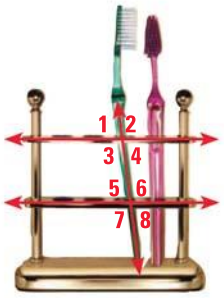
3. ★ **MULTIPLE CHOICE** In the figure at the right, which angle has the same measure as $\angle 1$?



Ⓐ $\angle 2$ Ⓑ $\angle 3$

Ⓒ $\angle 4$ Ⓓ $\angle 5$

USING PARALLEL LINES **Find the angle measure. Tell which postulate or theorem you use.**



4. If $m\angle 4 = 65°$, then $m\angle 1 =$?.

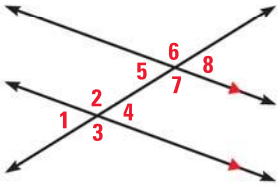
5. If $m\angle 7 = 110°$, then $m\angle 2 =$?.

6. If $m\angle 5 = 71°$, then $m\angle 4 =$?.

7. If $m\angle 3 = 117°$, then $m\angle 5 =$?.

8. If $m\angle 8 = 54°$, then $m\angle 1 =$?.

USING POSTULATES AND THEOREMS **What postulate or theorem justifies the statement about the diagram?**



9. $\angle 1 \cong \angle 5$

10. $\angle 4 \cong \angle 5$

11. $\angle 2 \cong \angle 7$

12. $\angle 2$ and $\angle 5$ are supplementary.

13. $\angle 3 \cong \angle 6$

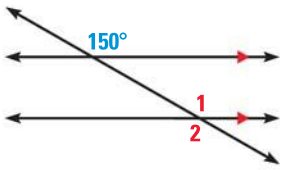
14. $\angle 3 \cong \angle 7$

15. $\angle 1 \cong \angle 8$

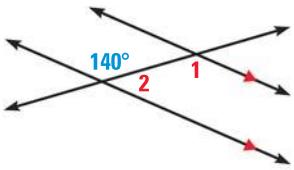
16. $\angle 4$ and $\angle 7$ are supplementary.

USING PARALLEL LINES **Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.**

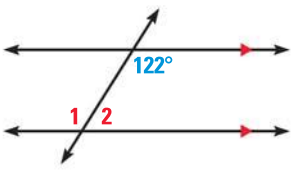
17.



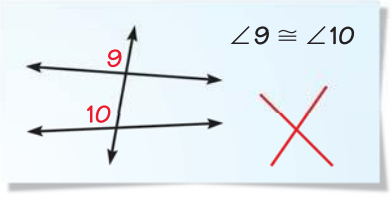
18.



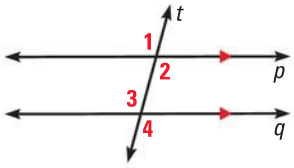
19.



20. **ERROR ANALYSIS** A student concludes that $\angle 9 \cong \angle 10$ by the Corresponding Angles Postulate. Describe and correct the error in this reasoning.

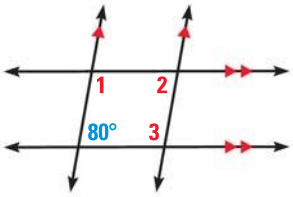


21. ★ **SHORT RESPONSE** Given $p \parallel q$, *describe* two methods you can use to show that $\angle 1 \cong \angle 4$.

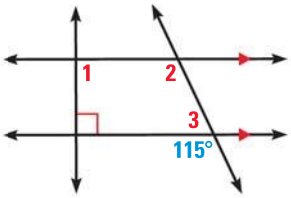


USING PARALLEL LINES **Find $m\angle 1$, $m\angle 2$, and $m\angle 3$. *Explain* your reasoning.**

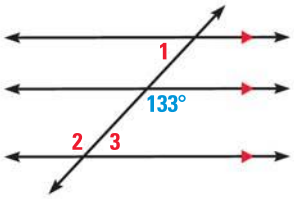
22.



23.



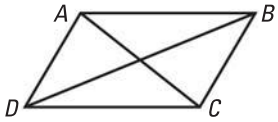
24.



ANGLES **Use the diagram at the right.**

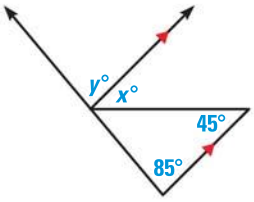
25. Name two pairs of congruent angles if $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel.

26. Name two pairs of supplementary angles if $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ are parallel.

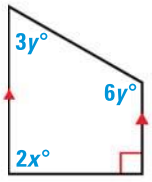


ALGEBRA **Find the values of x and y.**

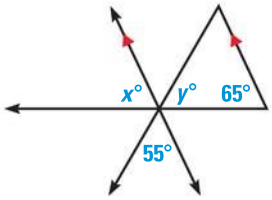
27.



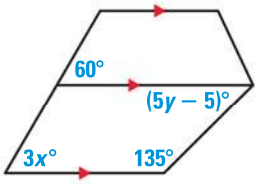
28.



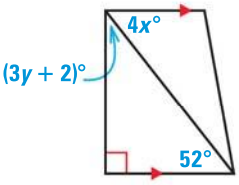
29.



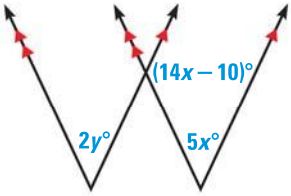
30.



31.



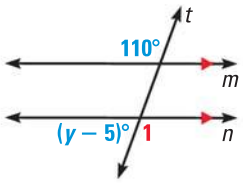
32.



33. ★ **MULTIPLE CHOICE** What is the value of y in the diagram?

Ⓐ 70 Ⓑ 75

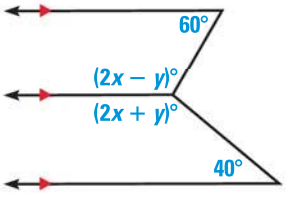
Ⓒ 110 Ⓓ 115



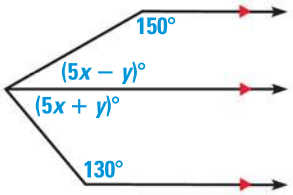
34. **DRAWING** Draw a four-sided figure with sides $\overline{MN}$ and $\overline{PQ}$, such that $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \parallel \overline{NQ}$, and $\angle MNQ$ is an acute angle. Which angle pairs formed are congruent? *Explain* your reasoning.

CHALLENGE **Find the values of x and y.**

35.



36.



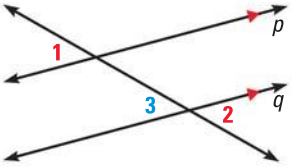
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 3 on p. 156 for Ex. 37

37. PROVING THEOREM 3.2 If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent. Use the steps below to write a proof of the Alternate Exterior Angles Theorem.



GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

a. Show that $\angle 1 \cong \angle 3$.

b. Then show that $\angle 1 \cong \angle 2$.

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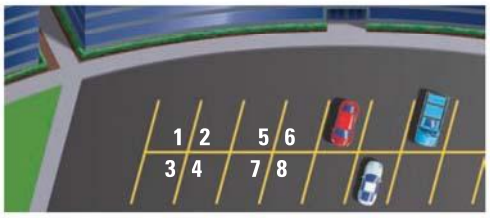
EXAMPLE 4 on p. 156 for Exs. 38–40

38. PARKING LOT In the diagram, the lines dividing parking spaces are parallel. The measure of $\angle 1$ is 110°.



a. Identify the angle(s) congruent to $\angle 1$.

b. Find $m\angle 6$.

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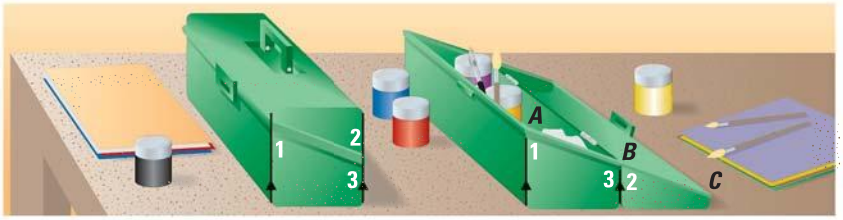
39. ★ SHORT RESPONSE The *Toddler*™ is a walking robot. Each leg of the robot has two parallel bars and a foot. When the robot walks, the leg bars remain parallel as the foot slides along the surface.

a. As the legs move, are there pairs of angles that are always congruent? always supplementary? If so, which angles?

b. *Explain* how having parallel leg bars allows the robot's foot to stay flat on the floor as it moves.

40. ★ EXTENDED RESPONSE You are designing a box like the one below.

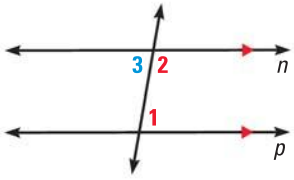


a. The measure of $\angle 1$ is 70°. What is $m\angle 2$? What is $m\angle 3$?

b. *Explain* why $\angle ABC$ is a straight angle.

c. What If? If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? *Explain*.

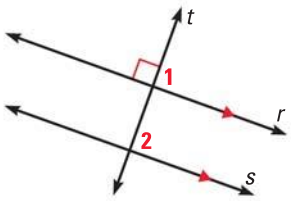
41. PROVING THEOREM 3.3 If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary. Write a proof of the Consecutive Interior Angles Theorem.



GIVEN ▶ $n \parallel p$

PROVE ▶ $\angle 1$ and $\angle 2$ are supplementary.

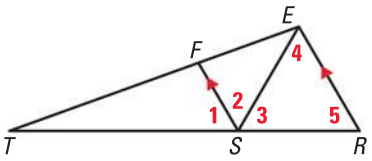
42. PROOF The Perpendicular Transversal Theorem (page 192) states that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other. Write a proof of the Perpendicular Transversal Theorem.



GIVEN ▶ $t \perp r$, $r \parallel s$

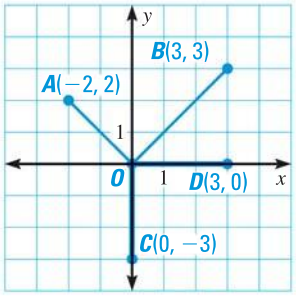
PROVE ▶ $t \perp s$

43. CHALLENGE In the diagram, $\angle 4 \cong \angle 5$. $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. *Explain* your reasoning.



MIXED REVIEW

44. Find the length of each segment in the coordinate plane at the right. Which segments are congruent? *(p. 15)*



Are angles with the given measures *complementary*, *supplementary*, or *neither*? *(p. 35)*

45. $m\angle 1 = 62°$, $m\angle 2 = 128°$

46. $m\angle 3 = 130°$, $m\angle 4 = 70°$

47. $m\angle 5 = 44°$, $m\angle 6 = 46°$

Find the perimeter of the equilateral figure with the given side length. *(pp. 42, 49)*

48. Pentagon, 20 cm **49.** Octagon, 2.5 ft **50.** Decagon, 33 in.

PREVIEW
Prepare for Lesson 3.3 in Exs. 51–52.

Write the converse of the statement. Is the converse true? *(p. 79)*

51. Three points are collinear if they lie on the same line.

52. If the measure of an angle is 119°, then the angle is obtuse.

QUIZ *for Lessons 3.1–3.2*

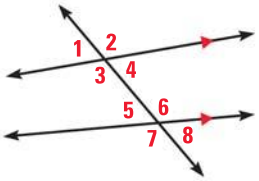
Copy and complete the statement. *(p. 147)*

1. $\angle 2$ and __?__ are corresponding angles.

2. $\angle 3$ and __?__ are consecutive interior angles.

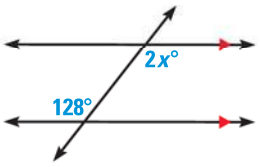
3. $\angle 3$ and __?__ are alternate interior angles.

4. $\angle 2$ and __?__ are alternate exterior angles.

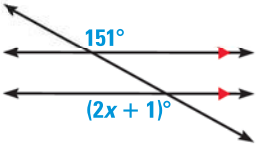


Find the value of *x*. *(p. 154)*

5.



6.



7.